Behaviour of RC Circuits

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Introduction

Understand the use and behaviour of RC circuit principles. Relate the solutions derived from the differential equations relating to the circuit and the observed behaviour using simulation. Gaining & recording insight associated with the observations.

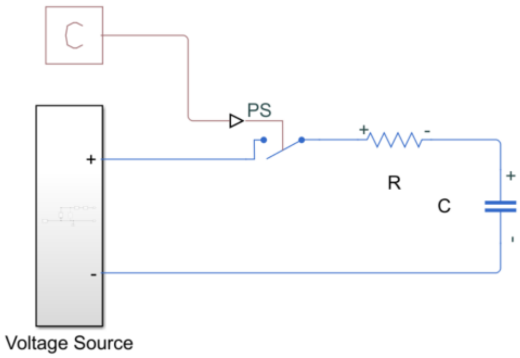
Learning Objectives

1. Model & analyze the behaviour of and RC circuit
2. Observe the bevaiour of and RC Circuit with switching circuits
3. Learn about the effect of parameter changes of the circuit with AC input
4. Observe and optimize component values for simple half-wave and full wave rectifier circuits

Background Theory

Derive Equations

Consider the following RC Circuit below



If the switch is closed we have the following relationship

$$V_s(t) = Ri(t) + \frac{1}{C}\int i(t)dt$$

or

$$V_s\delta(t) = R\frac{di(t)}{dt} + \frac{i(t)}{C}$$

```
syms i(t) Vs R C;
ode(t) = R*diff(i,t)+ i(t)/C == dirac(t)*C*Vs;
assume(t>0)
i(t) = dsolve(ode, i(0)== Vs/R)
```

i(t) =

$$\frac{\mathrm{Vs}\,e^{-\frac{t}{C\,R}}}{R}$$

assuming no charge on the capacitor initially

Voltage of the capacitor is then given by

```
V(t) = int(i(t), 0, t)/C
```

V(t) =

$$-\mathrm{Vs}\left(e^{-\frac{t}{C\,R}} - 1\right)$$

Plot representative current and voltage

```
Vo = subs(V, C, 1);
Vo = subs(Vo, R, 1);
Vo = subs(Vo, Vs, 1)
```

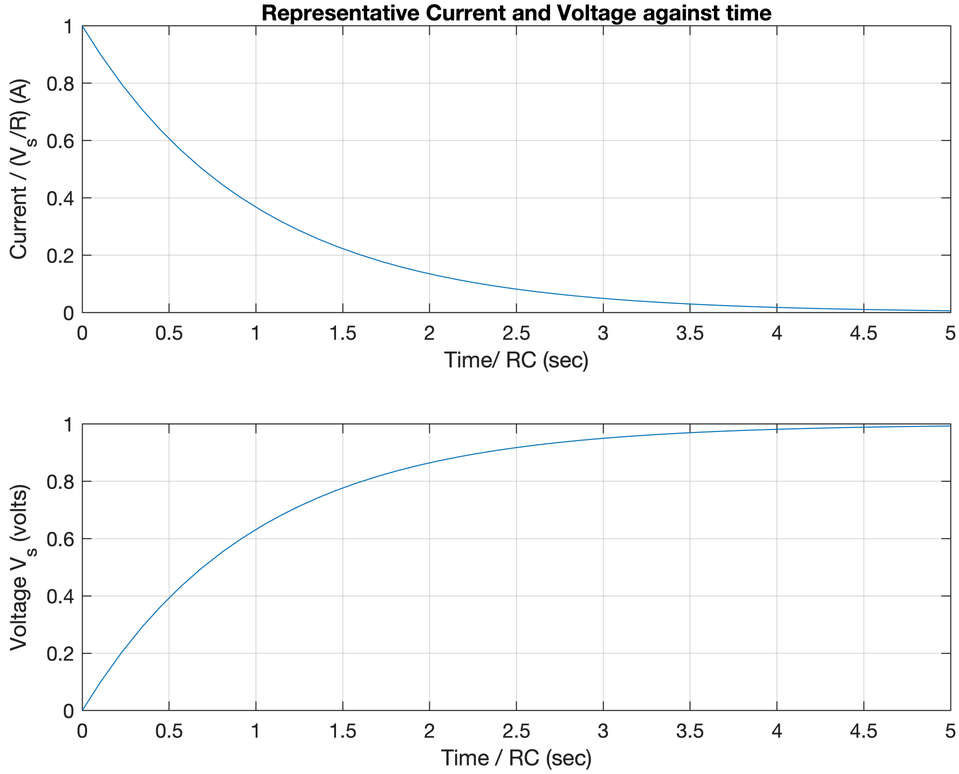
Vo(t) = $1 - e^{-t}$

```
io = diff(Vo,t)
```

io(t) = e^{-t}

Now plot representative current and voltage

```
subplot(211); fplot(io); grid on; title('Representative Current and Voltage against tim
subplot(212); fplot(Vo); grid on; xlabel('Time / RC (sec)'); ylabel('Voltage V_s (volts
```



Experiment 1: Effect of changing parameters

As you see we have the following two equations describing the behaviour of the system. We first create a MATLAB function from the symbolic expressions of voltage and current

```
Vc = matlabFunction(V)
```

```
Vc = function_handle with value:

    @(t,C,R,Vs)-Vs.*(exp(-t./(C.*R))-1.0)
```

```
Ic = matlabFunction(i)
```

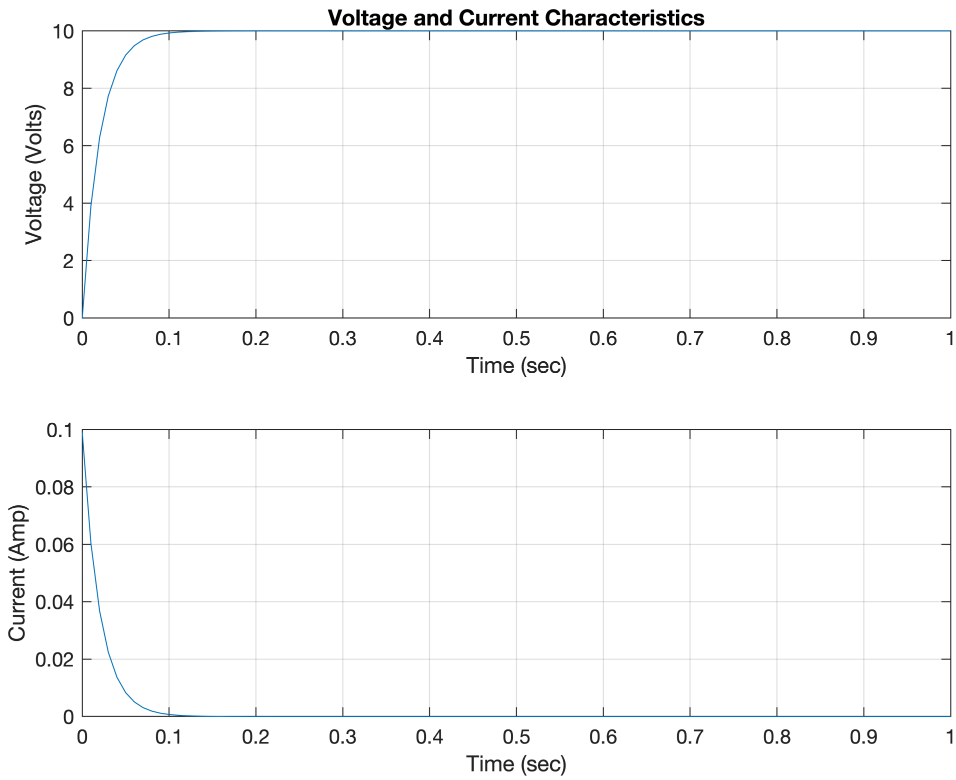
```
Ic = function_handle with value:

    @(t,C,R,Vs)(Vs.*exp(-t./(C.*R)))./R
```

Set some simple controls to modify resistance, capacitance and input voltage

```
Rc = 101; % Ohms
Cc = 0.0002; % Farads
Vsc= 10;
```

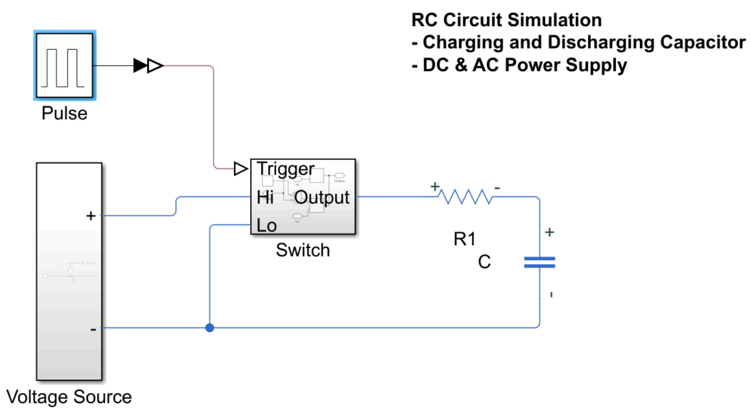
```
tc = 0:0.01:1;
figure;
subplot(211); plot(tc,Vc(tc,Cc, Rc, Vsc)); grid on; xlabel('Time (sec)'); ylabel('Volta
title('Voltage and Current Characteristics');
subplot(212); plot(tc,Ic(tc,Cc, Rc, Vsc)); grid on; xlabel('Time (sec)'); ylabel('Curre
```



Questions:

1. How does doubling or halving the value of R affect the voltage and current characteristics?
2. What about C?

Experiment 2: Basic RC - Charging and Discharging



1. Load & Run the Simulink Model
2. From the plots expermentally estimate the "Time Constant" of the Charging and Discharging

Experiment 2.1: Compare experimental and analytical model

Run the model to confirm similar behaviour as the analytical model

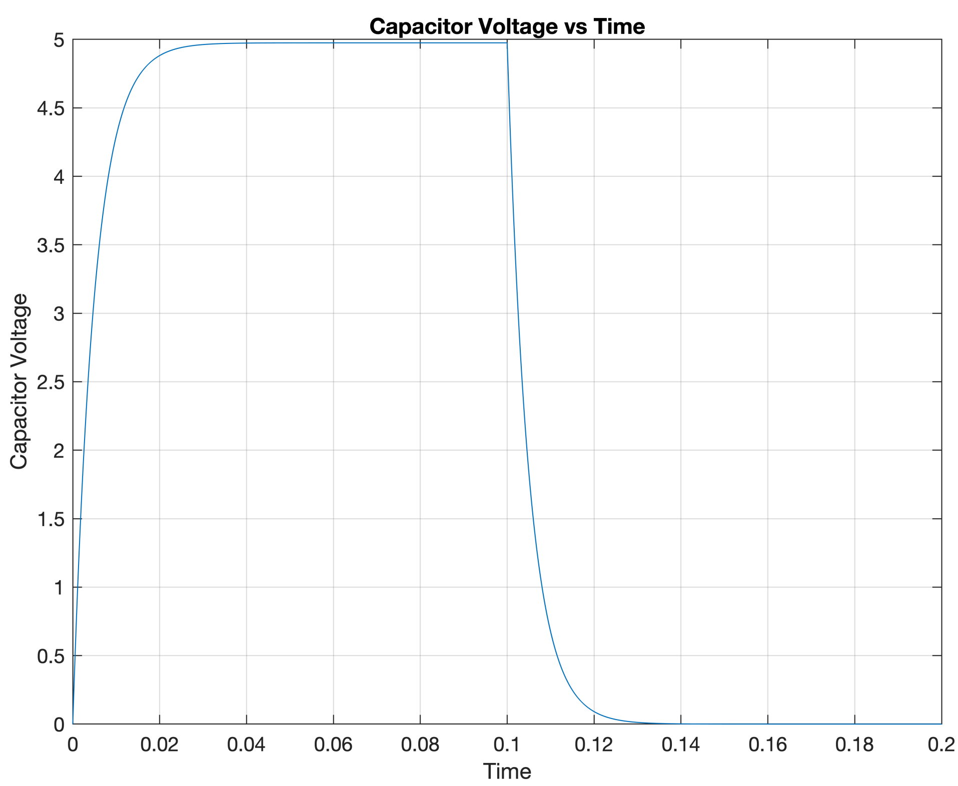
```
Vin = 10;                 % Volts
Rin = 1;                  % Ohms
SamplingFrequency = 5;    % Hz
C = 1e-4;                 % Farads
R1 = 100;                 % Ohms
R2 = 100;                 % Ohms
```

Open the simulation model

```
open_system('RC_Switching')
```

Change the values of R & C to confirm similar behaviour to the equations derived earlier

```
res_1 = plotCapVoltage();
title('Capacitor Voltage vs Time')
```



Experiment 2.2: Add components to the circuit

Add a 500 Ohm resistor across the capacitor & re-run the simulation. What is the time constant for charging and discharging? How do you explain the behaviour?

```
res_2 = plotCapVoltage();
title('Capacitor Voltage vs Time - Added 100 Ohm Resistor')
```

Comments about the effect of adding the load resistor

1. Max voltage dropped to 5 volts
2. Time constant halved

Experiment 2.3: Change switching frequency

Increase the frequency of the square wave by a factor of 10

```
res_3 = plotCapVoltage();
title('Capacitor Voltage vs Time - 50 Hz')
```

Comments:

Experiment 2.4: Increase switching frequency

Increase the frequency of the square wave by a factor of 10

```
res_4 = plotCapVoltage();
title('Capacitor Voltage vs Time - 500 Hz')
```

Comments:

1. What is the mean voltage at 'steady-state' ? Why?
2. How is the 'voltage ripple' related to the frequency of oscillation?

Experiment 3: Behaviour with alternating voltage source

Switch to AC supply. Note the behaviour of the circuit

```
SamplingFrequency = 1;
```

Predict the behavior with a 50Hz AC input.

What voltage do you expect to see at the load? Why?

```
res_5=plotCapVoltage();
title('Capacitor Voltage vs Time - 50 Hz AC Input')
```

Experiment 4: RC and rectifying circuit

Now add a diode between R1 & the voltage supply, Run the simulation.

```
res_6 = plotCapVoltage();
```

```
title('Capacitor Voltage vs Time - 50 Hz AC Input with Diode')
```

Experiment 4.1: Control voltage ripple

What value of capacitance gives you less than 0.05 volts ripple. What is the steady state voltage?

```
res_7 = plotCapVoltage();
title('Capacitor Voltage vs Time - 50 Hz AC Input diode < 0.05 V ripple')
```

Conclusions

Based on the discussions in the text book and these experiments, share 2 applications of RC Circuits and discuss why RC Circuits are the most appropriate choice for those applications.

Helper Functions

```
function res = plotCapVoltage()
clear out; res = sim('RC_Switching.slx');
pause(1);
figure;
plot(res.tout, res.simlog.C.v.series.values)
xlabel('Time'); ylabel('Capacitor Voltage'); grid on
end
```